Bounding the Residual Tropospheric Error by Interval Analysis

Jingyao Su, Steffen Schön
Institut für Erdmessung (IfE), Leibniz Universität Hannover, Hannover, Germany
Email: {suj, schoen}@ife.uni-hannover.de

IAG 2021 Beijing June 28–July 2 2021

1 Introduction

GNSS integrity monitoring requires proper bounding to characterize all ranging error sources. Unlike classical approaches based on probabilistic assumptions, the alternative interval-based integrity approach depends on deterministic interval bounds as inputs [1]. Different from a quadratic variance propagation, the interval approach has intrinsically a linear uncertainty propagation which is adequate to describe remaining systematic uncertainty [2].

As one of the primary error sources, the tropospheric error is corrected in practice widely by the well-developed empirical troposphere models. In order to quantify and bound the residual tropospheric error for those models, we will make a proposal on how to derive the required interval bounds based on interval analysis.

2 Methodology: Sensitivity Analysis

The sensitivity analysis of the tropospheric correction models is first implemented via interval arithmetic. The resulting sensitivities, together with carefully estimated uncertainties of model influence factors, are used to construct deterministic intervals accounting for the uncertainty of the tropospheric correction model. The following shows the reformulation of the classical sensitivity analysis [3] in the context of intervals.

- **In view of interval approach:**

Model's sensitivity (S_f) to each factor (p_i) is derived by the set image of the model (f) when the dedicated factor ($p_i = p^*$) is an interval-value ($[p_i] = p^* + [\underline{\Delta_{p_i}}, \overline{\Delta_{p_i}}] = [\underline{p_i}, \overline{p_i}]$):

$$S_f(p_i) = f([p_i]) - f(p^*)$$

where $f(p^*)$ is the model's output under a certain condition (p^*). Model's uncertainty budget ($[\underline{\Delta_f}, \overline{\Delta_f}]$) can be expressed as sum of all factors' contributions:

$$[\underline{\Delta_f}, \overline{\Delta_f}] = \sum S_f(p_i)$$

which is a deterministic interval with a lower bound ($\underline{\Delta_f}$) and an upper bound ($\overline{\Delta_f}$). It is not necessarily symmetric.

3 Implementation of the Saastamoinen Model

Special care must be paid to the uncertainty budget of meteorological parameters. To this end, long-term statistics against on-site measurements are performed to estimate the interval bounds of meteorological parameters that are needed as input to the tropospheric correction models.

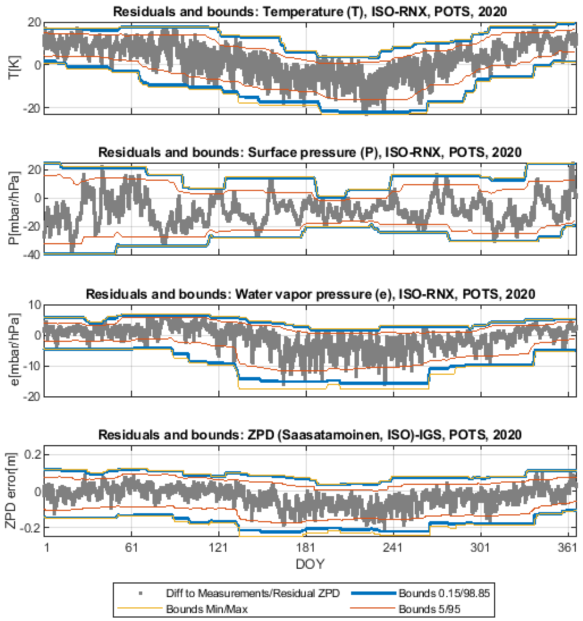

Figure 1. Example results of IGS station POTS in 2020: residuals (ISO-to-RNX) and bounds for meteorological parameters from long-term statistics. ZPD bounds are computed with these results, and compared to residuals (Saastamoinen with ISO-to-IGS ZPD)

The dense network of Deutscher Wetterdienst (DWD) facilities the analysis for multiple stations and the estimation for the geographical distribution of error bounds over the country by means of interpolation. Example results cf. Fig. 2 and Fig. 3.

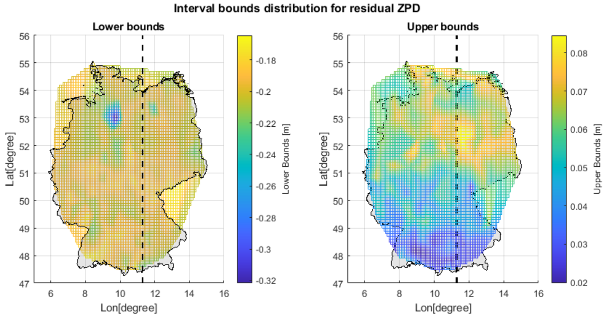

Fig. 2. Example results: geographical distribution of lower bounds (left) and upper bounds (right) for residual ZPD error over Germany on DOY 239 in 2020.

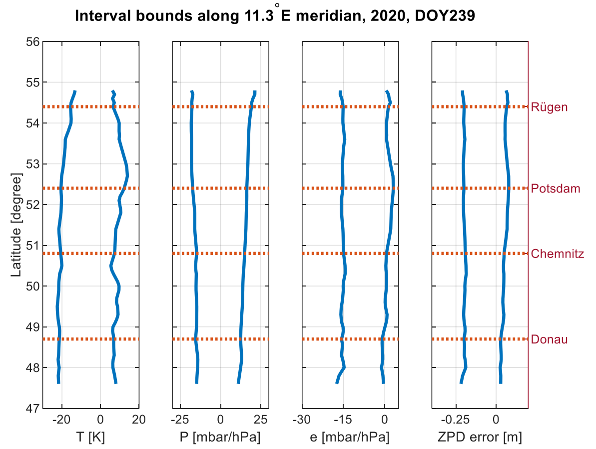

Fig. 3. Example results: Interval bounds for meteorological parameters (from ISO-to-DWD) and ZPD (Saastamoinen with ISO) along the 13.1°E meridian (denoted in Fig. 2 as black dashed line) on DOY 239 in 2020. Climate data source: Deutscher Wetterdienst (DWD) .

4 Validation by Ray-Tracing and Measurements

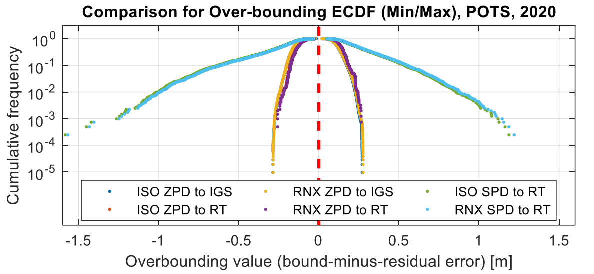

Fig. 4. Example results: ECDF of over-bounding values (bound-minus-residual error) for residual SPD/ZPD from Saastamoinen model with standard atmosphere (ISO), as well as on-site measurements (RNX) at POTS in 2020. The sampled ZPD/SPD estimates from Vienna Ray-Tracer (RT) [4] serve as reference in addition to the IGS ZPD products.

5 Conclusions and Outlook

- The implementation of the Saastamoinen model indicates the interval method's feasibility to qualify and bound residual tropospheric errors.
- Uncertainties of the model influence factors must be carefully estimated. Estimation for meteorological parameters is done through long-term statistics against on-site measurements.
- Further work will focus on the potential impact of the mapping functions, as well as the implementation of other empirical tropospheric correction models.

Reference

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[3] S. Schön & H. Kutterer, Realistic Uncertainty Measures for GPS Observations, 2005
[4] A. Hofmeister & J. Böhm, Application of ray-traced tropospheric slant delays to geodetic VLBI analysis, 2017

Acknowledgement

This work was supported by the German Research Foundation (DFG) as part of the Research Training Group i.c.sens [RTG 2159]. The authors gratefully acknowledge Deutscher Wetterdienst (DWD) for providing climate data, Helmholtz-Centre Potsdam GFZ German Research Centre for Geosciences for providing measurement data of POTS station and IGS for providing GNSS data and products.

Leibniz Universität Hannover